PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Determination of the Yard in Wave-Lengths of Light

A. E. H. Tutton

Phil. Trans. R. Soc. Lond. A 1932 **230**, 293-322
doi: 10.1098/rsta.1932.0008

VIII. *Determination of the Yard in Wave-lengths of Light.*

By A. E. H. Tutton, *M.A.*, *D.Sc.*, *F.R.S.*

(Received March 19, 1931.—Revised June 12, 1931.—Read April 30, 1931.)

The remarkable research carried out in the year 1892 by Professor Albert A. Michelson, the full results of which were published in 1894,* in which he evaluated the International metre in terms of the wave-lengths of the red, green, and blue light radiations of cadmium, at the Bureau International des Poids et Mesures in Paris (Sèvres), provoked great interest in this new departure in metrology. For it is an obvious advantage that a standard of length shall be based upon a definite and invariable (at a suitable standard temperature and pressure) physical constant, such as the wave-length of the light radiation corresponding to a single unresolvable line of the spectrum, which can be absolutely reproduced at any time, rather than upon the length of a metallic bar, subject to all the vicissitudes of time and circumstance.

As one important result of his own investigations, Professor Michelson had shown that in the red line Cd_r of incandescent cadmium vapour we possess such a suitable standard of wave-length, this line resisting resolution by even the echelon spectroscope, and to-day it is the recognised standard wave-length, the exact value of which (see p. 318) was fixed by the very same research in which he evaluated the metre. These measurements of so refined a character were only possible by the use of the delicate interferometer which he had devised and which now so familiarly bears his name.

Another form of interferometer was, however, subsequently devised by Messieurs Ch. Fabry and A. Perot, and in the year 1906 they, in collaboration with Monsieur J. R. Benoit, Directeur of the Bureau International, carried out at the Laboratoire d'Essais du Conservatoire des Arts et Métiers in Paris, of which M. Perot was Directeur, a redetermination of the number of wave-lengths of red cadmium light in the metre, the results of which were published in brief form in 1907,† and in full in 1913.‡ Their final result was practically identical with that of Professor Michelson, the concordance being within a single wave-length, the Michelson value being 1,553,163·50, and that of Benoit, Fabry, and Perot 1,553,164·13.

* 'Travaux et Mémoires du Bureau International des Poids et Mesures,' vol. 11 (1894).
† 'Comptes Rendus,' vol. 144, p. 1082 (1907).
‡ 'Trav. et Mém. du Bur. Int.,' vol. 15, p. 3 (1913).

 [Published September 22, 1931.

But these beautiful determinations, practically perfect in their interferometry, and however concordant the results appear, are subject to an important drawback, that the finest engraved defining marks on the standard bars themselves, which define the limits of the metre, contain in their thickness the equivalent of no less than 15 interference band transits. The 1902 platinum-iridium British yard, of "Tresca" section (a modified H-shape), has similar defining lines to the prototype metre, which were engraved at the Sèvres Bureau by M. BENOIT, and the author has personally measured their thickness in bands, which were 15 in number. The prototype British Imperial Standard yard, of Baily's metal, of 1 inch square section and 38 inches long, has defining lines engraved by Messrs. Troughton and Simms, the makers, on gold studs let into wells near each end so as to be placed at the centre of the bar-depth, and these defining lines allow as many as 45 interference bands ($22\frac{1}{2}$ wave-lengths) in red light of hydrogen or cadmium to pass while traversing their thickness, although their edges are much sharper than those of the metre or platinum-iridium yard, which are very ragged.

Thus after all the refined interferometry has been completed, the final comparison has to be made with the interval between two such defining lines as have been described, and although special devices and immense arithmetical labour were expended in getting as accurate a result as possible for this final lap of the work, the results, as Professor MICHELSON himself remarks in his book,* are nevertheless subject to this unavoidable defect. In fact, the results cannot obviously be valid to a single wave-length, let alone to one or two decimal places.

In an appendix to the memoir on the Wave-length Comparator which the author devised for the Standards Department and described to the Royal Society in 1909,† attention was called to some remarkable diamond-engraved wave-length rulings, lines 1/40000-inch apart on speculum metal, which had been made both for the Standards Department and for the author, by the late much regretted Professor H. J. GRAYSON, of Melbourne University. Professor GRAYSON, as is well known, had ruled lines with extraordinary clearness as close as 120,000 to the inch, but for the practical purposes of fine measurement his lines of 40,000 to the inch are pre-eminently suitable. For the 1/40000th of an inch is the wave-length of red light (H_α being 1/38703 inch and Cd_r 1/39450 inch). They are as clear and sharp as spider-lines, apparently at least a millimetre apart, when viewed in the comparator microscopes, or that of the author's interferometer, using the special high power dry objectives provided. It was pointed out what an ideal fiducial mark the central one of five such diamond-ruled parallel lines afforded; and such a group of five lines at 1/40000th of an inch apart, ruled by Professor GRAYSON on a small slab of highly polished speculum metal, and covered for protection with a thin cover-glass cemented by realgar or balsam, has since been used by the author as a "location-signal" (a convenient term for such a fiducial mark) in many of his interferometric measurements, the third, central, line being regarded as *the* fiducial

* "Light Waves and their Uses," p. 100 (Univ. of Chicago Press, 1903).

† 'Phil. Trans.,' A, vol. 210, p. 28 (1909).

mark or position-defining line. Their appearance in a typical field is shown in fig. 1.

Besides the five important lines themselves, each location-signal also bears two stronger "finder" lines, parallel with and a little away from the five, and also two similar stronger lines at right angles to the seven, so that the centre of the rectangle which the four thicker lines make where they cross one another may be taken as the centre of measurement, easily recoverable for repetition purposes. The single thin horizontal line, and the pair of parallel vertical ones, shown on the left of the five in fig. 1, are the mutually adjustable spider lines of the microscope. If some physical movement, such as the piezo-electric movement of a crystal, for instance, is to be measured on the interferometer, it is only necessary to cause the part which moves to push along a little slider carrying the speculum ruling, when the amount of movement can be accurately measured in interference bands by following up, at leisure, the movement with the microscope; that is, by focussing with the microscope (which itself carries on its side parallel to its axis one of the interference-producing glass surfaces) the location-signal before and after the movement, and while thus moving from the one position to the other (in following up the movement with the travelling microscope) by counting the interference bands as they pass the reference-centre of the interferometer telescope.

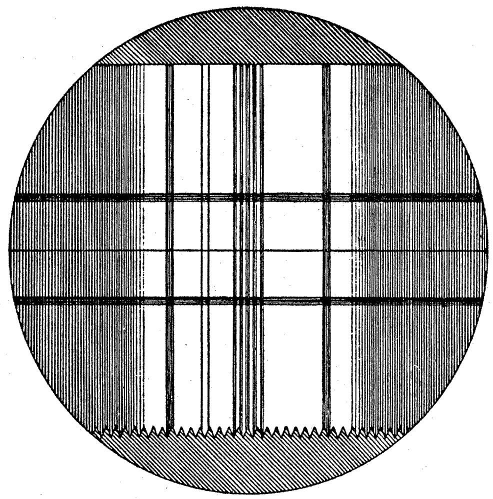

FIG. 1.—The Location-signal.

In addition to these single location-signal rulings on small slabs of speculum metal about 10 by 9 mm. in size, and just over 2 mm. thick, as shown in fig. 2, Professor GRAYSON was also kind enough to make, with his unique and now unhappily no longer available skill, three scales, marked A, B, and C, of $1\frac{1}{8}$ inch between the two end location-signals, on slabs of speculum metal in the cases of A and B, and of speculum on glass in the case of C, the same (9 mm.) width and thickness (2 mm.) as the single ones but naturally somewhat more than $1\frac{1}{8}$ inch long. One of them, A, is represented in fig. 3.

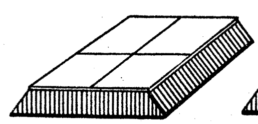

FIG. 2.—A Single Location-signal.

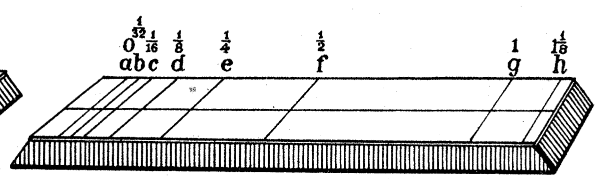

FIG. 3.—$1\frac{1}{8}$-inch Scale of Location-signals.

These scales are divided at the following points; zero, $\frac{1}{32}$-inch, $\frac{1}{16}$-inch, $\frac{1}{8}$-inch, $\frac{1}{4}$-inch, $\frac{1}{2}$-inch, 1-inch, and an extra $\frac{1}{8}$ inch after the 1-inch mark in the cases of A and C, but before the zero in the case of B, so that the extreme limiting marks of this scale are $1\frac{1}{8}$ inch apart. Each of these division marks, however, which to the naked eye or a low microscope-power appears to be a single line, is in reality a location-signal, as shown in fig. 1. The longitudinal finder lines are ruled along the whole length of the slab, centrally with respect to the long edges of the latter, with which they are parallel.

The accuracy with which Professor GRAYSON spaced his rulings is a source of amazement to the author. For not only do the four intervals between the five lines always correspond to the passage of eight interference bands in the interferometer, the bands corresponding to half wave-lengths, when one is using red hydrogen or red cadmium light to produce the bands, but the spacing of the intervals between the location-signals is so accurate that, for instance, one of his $\frac{1}{32}$-inch intervals taken at random corresponds to the passage of 2712 bands in yellow neon light, within a single band of what it should be. Obviously such location-signals, the central line of each of which can be accurately allocated to the spider-lines of the microscope to a tenth of a band, and the thickness of which line itself is well within the passage of a single band, offer unique advantages for metrological purposes. The lines are wonderfully clearly focussed when one uses a bluish green light, reflected by the ordinary opaque-object viewing mirror (a thin cover-glass plate serves best as reflector) placed just above the objective; as source, a pointolite lamp serves admirably, at a considerable distance away (to avoid its heating effect) and provided with a copper-sulphate solution colour filter. The fallacy that anything of wave-length size or less should be invisible, has, of course, been quite exploded by the Grayson rulings.

In the appendix to the comparator memoir (*loc. cit.*, p. 31) it was suggested that such Grayson-ruling location-signals, if engraved on a standard bar of platinum-iridium or invar at suitable intervals, could be used instead of the glass étalons of MICHELSON or FABRY and PEROT, for a simpler and more direct method of determining the total number of wave-lengths of a standard radiation, such as red cadmium light, comprised in the yard, counting as a base-line the interference bands, about 2500, in the $\frac{1}{32}$ inch. Professor GRAYSON made several experimental rulings on metals other than speculum, namely, platinum-iridium, gold, silver, Baily's metal, and invar, but none proved so clear, and especially so sharp and without chipping at the edges, as those on polished speculum. It should expressly be remarked that Professor GRAYSON himself truly planed and polished all the speculum surfaces on which he ruled his lines. It was, therefore, proposed to use the speculum rulings, and to mount them immovably on an invar bar. At this time, however, the author's official removal from London to the western district caused the work at the Standards Department to be broken off, and since then the author's crystallographic researches occupied all his spare time. At length, however, the programme of crystal work being completed, and his retirement to Cambridge having occurred, this interesting metrological problem has been resumed. A great assistance, moreover, has in the meantime accrued by the construction of the universal interferometer, which the author described to the Royal Society in 1923.* This instrument embodies the essentials, and with improvements, of that half of the comparator which includes the interferometer and its connected right-hand microscope. It has enabled the author, in the quietude of his private laboratory in Cambridge, to become more and more familiar with the beautiful rulings of GRAYSON, and to practice counting as many interference bands as possible, with the view of adopting a counted base-line much longer than has hitherto been adopted or considered possible. That of MICHELSON, comprising his smallest étalon, was of 1212 bands in red cadmium light and (the maximum number counted) 1626 in the blue radiation of cadmium.

For a long time the 2400–2700 bands in the $\frac{1}{32}$-inch proved too many for eye endurance. But prolonged familiarity and practice has at length rewarded the author by enabling him to count not only this quantity but the number in double the length, the approximately 4800 (red) and 5400 (yellow neon light) interference bands in the $\frac{1}{16}$-inch, an operation which occupied $1\frac{1}{4}$ to $1\frac{1}{2}$ hours. Indeed, as this has been achieved many times over for this interval on both the A and B slabs (and also on slab C for confirmation purposes), and the comparator enables that on either of them to be transferred alongside the other to the similar (second) interval of like length, by comparison and measuring of any difference in bands, the total number of bands in the $\frac{1}{8}$ inch, about 9670 in red and 10,850 in yellow light, have actually been counted, and in duplicate, for both A and B scales. This, then, may be regarded as the base-line of the present determination. This base-line work was carried out at a temperature practically exactly 62° F., the official temperature, the limiting values being 61·8°–62·2°, the author's laboratory being readily maintained in the neighbourhood of 62° for hours during the months of August, September and October.

After the completion of this base-line work at Cambridge, the rulings were brought to the Standards Department at Old Palace Yard, mounted on an invar bar, and the remainder of the research has been completed there on the Tutton Comparator, the whole room in which this is housed being thermostatically (electrically) maintained at 62° F., as described in the comparator memoir (*loc. cit.*, p. 10).

By the kindness of the Superintendent of the Metrology Department of the National Physical Laboratory, a standard yard invar bar, No. 27, has been available, the very low thermal expansion of which is

$$L_t = L_0 (1 + 0{\cdot}000\ 001\ 12t + 0{\cdot}000\ 000\ 000\ 16t^2).$$

It is of H-section, and on one of its flat side faces the rulings have been mounted, with hard Canada balsam which had been warmed over a low flame for days and then dissolved in benzene. The $1\frac{1}{8}$-inch scales were first mounted, one at either end, so that the zero location-signal of A and the last location-signal of B formed the left and right

* 'Proc. Roy. Soc.,' A, vol. 104, p. 47 (1923).

limits respectively of the yard. This adjustment was made with the greatest care, so that the rulings were absolutely perpendicular to the length of the bar, and symmetrical to the bar-sides, so that the longitudinal finder-ruling in each case was central, and so that the yard thus made was as near as possible to the Imperial Standard Yard in length. In doing this the standard yard of 1926 was happily available for reference and comparison, the exact relation of which to the Imperial prototype yard had only recently been most carefully determined and officially recorded.

The single location-signals were then also cemented into their designed positions, one at $2\frac{1}{4}$ inches from each end, another pair at $4\frac{1}{2}$ inches from the two ends, yet another pair at the 9-inch positions from each end, and finally one in the middle at 18 inches. The various location-signals were each labelled with a letter, for purposes of identification, as shown on fig. 4, so that they may be easily recognised in the tables on pp. 307–315.

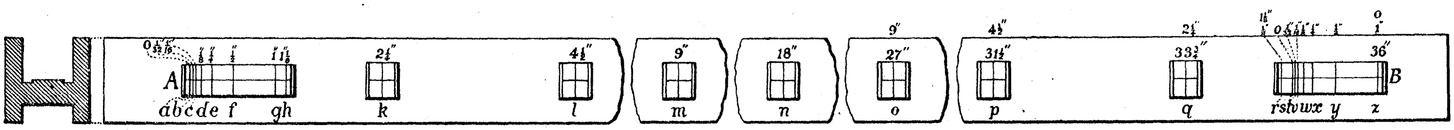

Fig. 4.—The Invar Bar with Rulings in position.

It will be observed that the first single location-signal from each end, *k* or *q*, is situated at twice the length from zero or 36 inches of the $1\frac{1}{8}$-inch ruling, but at the same distance ($1\frac{1}{8}$ inch) from *h* or *r*; the next, *l* or *p*, is at double that distance from zero or 36 inches, $4\frac{1}{2}$ inches, but the same from *k* or *q*, $2\frac{1}{4}$ inches; the next, *m* or *o*, at 9 inches from the zero or 36-inch mark, is at $4\frac{1}{2}$ inches from *l* or *p*; and the middle ruling *n*, 18 inches from zero or the 36-inch ruling, is also at 9 inches from *m* or *o*.

This arrangement ensures that there are two equal intervals of each dimension adjacent to each other throughout the whole yard; and it enables any interval on one half of the scale to be compared with either of the two similar ones on the other half, by means of the comparator and the interferometer attached to its right-hand microscope. It was this arrangement which enabled the base-line of counted bands to be made $\frac{1}{8}$-inch, and not merely $\frac{1}{16}$-inch, the two adjacent sixteenths being added together. The $1\frac{1}{8}$-inch ruling C is not shown in fig. 4, to save confusion. It was mounted somewhat to the right of the 18-inch signal.

The Radiations employed.—The radiations which the author has relied on in the investigation are those of the red line of hydrogen H_{α}, and especially those of the yellow line of neon, Ne_{c}. The wave-length of the latter has been very accurately determined (see p. 318) by several recent workers, with reference to the standard wave-length of red cadmium light, Cd_{r}. The wave-length of hydrogen has been the source of some controversy, Fabry and Perot's well-known curve (the measurements for which did not extend so far into the red as H_{α}) by extrapolation probably over-correcting Rowland's most carefully determined value. But as its exact value has been one of the results of the present investigation, which is in fairly close agreement with the best of recent determinations, this difficulty has largely disappeared (see p. 319).

Both neon and hydrogen give magnificent interference bands in the author's interferometer and comparator, the field being large (apparently about four inches in diameter), and the upright rectilinear well separated bands (about a dozen in the field affording a convenient width) are particularly suitable for micrometric measurement. Of all sources hydrogen gives the finest bands the author has ever experimented with, when the GUILD form (H-shaped with a large bulb, of 1 litre capacity at least, blown on one limb) of vacuum tube is used, with an induction coil or transformer to actuate it. And as only a thin film of air, usually less than a millimetre thick, and never exceeding 2½ mm., is used between the two glass surfaces (one carried by the microscope itself) reflecting the interfering light rays, involving no such large order of interference as in the methods of MICHELSON or FABRY and PEROT, it is admirably suited for these determinations. Neon is almost as good, but, unlike hydrogen, which has so fortunately no line anywhere near the red H_α, one has to be very careful to screen off all light from other neighbouring lines, both by restricting the rectangular aperture of the little reflecting prism at the autocollimating focus of the interferometer telescope to a mere slit (which is the real source of the interfering light), and by drastic use of the iris diaphragm, in order to avoid disturbance (obscuring of the bands) by secondary unwanted interference.

As regards cadmium light, the great drawback that the vacuum tube carried by the side tube of the interferometer telescope requires to be heated to 320°–340° C. in order to provide the light has decided the author, after many trial experiments, not to rely upon any determinations carried out with it, or with a cadmium arc (kindly lent by Professor LOWRY) which was also tried. For the heating effect is destructive to accuracy in the determinations. It is far better to calculate the number of wave-lengths of the standard red cadmium radiation in the yard from the irreproachable neon results, and to confirm the result by calculation from the determinations with red hydrogen light, the line H_α being so close to the cadmium line Cd_r. The concordance of the two calculations will be shown in the sequel to prove the wisdom of this decision.

The Counting of the Base-line.—This was carried out as follows. The 1⅛-inch scale, say A, was arranged under the high power of the microscope, so that the location-signal marking the zero position was in focus in the brilliant greenish-blue light from the pointolite lamp (with copper sulphate solution filter-cell), and with the five fine rulings truly parallel to the pair of spider-lines, and the centring rectangle of the four finder lines in the middle of the field. The objective was a dry ⅙-inch combination, used with a high-power eyepiece, calculated to afford a clear, safe, working distance between the objective and the cover glass of the ruling. A good field of interference bands was arranged to be visible in the interferometer telescope, the parallel straight lines of the bands being adjusted so as to be of convenient width and separation and truly vertical, parallel to the pair of measuring spider-lines of the micrometer eyepiece of the telescope, which would have been adjusted for their separation to the size of the little silvered ring, the centre of reference, shown in fig. 5. The microscope was then finally adjusted by the big wheel which drives it, so as to bring the middle ruling of the five to the centre of the spider-lines, which have been adjusted once for all just to include the central ruling (or any one line) of the five within their width. Then the interference bands are observed, and if one is not exactly centred, the initial fraction of a band is measured. The movement of the microscope over the interval between this location-signal and the next can then be commenced, by slow rotation of the big wheel, counting the bands as they pass steadily through the silver-ring reference centre and pair of spider-lines, and adjusting the speed so as to secure deliberate counting. To miss a band or make a single error in counting is, of course, fatal. A very helpful fact, however, is that if one rests the ball of the hand on the top of the pedestal on which the big driving wheel is mounted, and which is fortunately so shaped as to make a comfortable rest, and effects the rotation of the wheel with the first finger and thumb, it so happens that one can only pull the wheel over sufficiently to bring one more band to the centre, so that two bands at a time in rapid succession can never pass and cause any missing of a band. One band *per* second is a very suitable speed, both safe and not tiring. After the passage of 100 bands, the fact is noted in one's notebook, and each 100 as it arrives is duly given its proper sequence number. As the end of the interval approaches one slows down, for a look has to be given to the microscope; the eye can safely be taken momentarily from the interferometer telescope to do so, as the bands immediately stop moving when the big wheel is arrested, at most half a band of further movement occurring. This slight backlash is not a defect, but due to a minute amount of spring possessed by the steel-coil shaft, which is used in preference to a solid one to connect the big wheel with the driving worm, which actuates the worm wheel-head of the very accurate screw which moves the microscope-carrying block, such a coiled shaft being free from disturbing thrust. Eventually, when we are entering on the last 100 bands, the next location-signal comes in sight in the field of the microscope, and we can go as slowly as necessary, with momentary and intermittent transference of the eye from interferometer to microscope, until at last the middle ruling of the five is duly centred between the spider-lines. The reading of the last fraction of a band (if none is actually arrested at the exact centre of reference) can then be taken, and the operation is completed. At the rate of 1 per second 3600 bands would, of course,

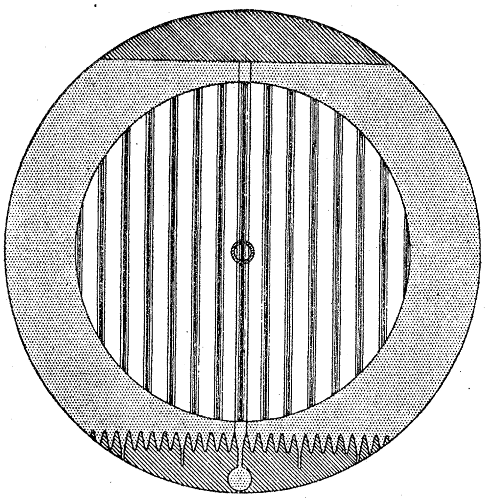

FIG. 5.—The Field of Interference Bands.

pass the centre in an hour; the author usually completed the 4835 for the $\frac{1}{16}$-inch in red hydrogen light in $1\frac{1}{4}$ hours. For the 5423 bands in yellow neon light about $1\frac{1}{2}$ hours were taken. This is a long period in which to have to concentrate one's observation without a break, and naturally the eye fatigue was always very considerable; but perseverance in practice enabled one at length to achieve the desired result many times repeated. The temperature and barometric readings were, of course, taken just before, and again just after, the observation. In all the determinations the bands remained upright and of similar thickness and spacing throughout.

In determining the bands in the first $\frac{1}{16}$-inch interval one comes midway to the location-signal ruled at the $\frac{1}{32}$-inch, and one can rest there a few moments while the reading of the band position is taken, for the adjustment of that location-signal to the spider-lines. In yellow neon light one can then go on at once with the second half of the $\frac{1}{16}$-inch interval, for this addition to the thickness of the air-film between the two glass plates (one of them carried by the microscope itself) which are reflecting the interfering light makes no difference to the intensity of the bands. One starts always, however, this base-line determination with a very close approximation of the two glass surfaces, say something less than a millimetre of separation; then, at the end of the measurement, the thickness of the air film will still only be a little over 2 mm. In the case of red hydrogen light, however, at this latter thickness the band intensity begins to become diminished, but is still adequate for accurate observation. One can, however, avoid the defect altogether by making two operations of the two $\frac{1}{32}$-inch intervals comprised in the $\frac{1}{16}$-inch, starting the second as well as the first with a 1 mm. or less separation of the glass plates. Both methods have been adopted in different determinations, and with equally satisfactory results. The loss of intensity is, of course, due to the line H_α being really a very close doublet (see p. 319), the diminution in intensity of the bands being a preliminary to disturbing secondary interference. As the author has never used hydrogen light for any intervals beyond the 2 mm. just referred to, this secondary interference has never approached the importance when it might have any influence on the results.

The results of the counts in both hydrogen H_α and neon Ne_c light now follow (pp. 302 and 303).

The corrections for temperature and pressure are negligible, being much less than a tenth of an interference band. For the discussion of temperature corrections see p. 306.

The final results of the counts of interference bands in red hydrogen light and at 62° F., for the initial $\frac{1}{16}$-inch intervals on the three respective ruled scales A, B, and C, are thus :—

For scale A, ac = 4832·7

,, B, sv = 4835·8

,, C, $\beta\delta$ = 4835·8.

The facts that the $\frac{1}{16}$-inch intervals for scales B and C were identical, and that both were 3·1 bands larger than the corresponding interval on scale A, were confirmed several times over by direct comparison on the wave-length comparator.

The Counted Basal Intervals, $\frac{1}{16}$-inch.—Determinations in Red Hydrogen Light (H_{α}, line C).

Number of interference bands in first $\frac{1}{32}$-inch interval.	Number of interference bands in second $\frac{1}{32}$-inch interval.	Total interference bands in whole $\frac{1}{16}$-inch interval.	Temperature limits.	Barometric height.	Correction for t and p.
		Scale A.			
ab	*bc*	*ac*	° F.	mm.	
2414·6	2417·5	4832·1	62·0–62·1	760·2	Nil
2415·1	2417·3	4832·4	61·8–62·1	756·2	"
2416·5	2417·0	4833·5	62·0–62·2	759·0	"
2415·5	2417·1	4832·6	62·0–62·0	755·6	"
2416·3	2416·6	4832·9	61·9–62·0	751·6	"
2415·6 mean	2417·1 mean	4832·7 mean	62·0 mean		
		Scale B.			
st	*tv*	*sv*			
2416·9	2418·5	4835·4	62·0–62·1	760·7	Nil
2418·0	2417·8	4835·8	62·1–61·9	764·0	"
2418·8	2418·0	4836·8	62·1–62·0	760·7	"
2417·8	2417·8	4835·6	62·0–62·1	755·6	"
2417·5	2417·9	4835·4	61·9–62·0	756·9	"
2417·8 mean	2418·0 mean	4835·8 mean	62·0 mean		
		Scale C.			
βγ	γδ	βδ			
2417·6	2418·6	4836·2	62·0–62·1	760·7	Nil
2417·3	2417·7	4835·0	62·1–62·0	762·0	"
2417·3	2418·4	4835·7	62·0–62·0	759·0	"
2417·4	2417·9	4835·3	62·1–62·1	753·1	"
2417·8	2418·8	4836·6	61·8–62·0	759·0	"
2417·5 mean	2418·3 mean	4835·8 mean	62·0 mean		

Determinations in Yellow Neon Light (wave-length 5852).

Number of interference bands in first $\frac{1}{32}$-inch interval.	Number of interference bands in second $\frac{1}{32}$-inch interval.	Total interference bands in whole $\frac{1}{16}$-inch interval.	Temperature limits.	Barometric height.	Correction for t and p.
		Scale A.			
ab	*bc*	*ac*	° F.	mm.	
2709·8	2710·3	5420·1	61·8–62·2	753·1	Nil
2709·5	2710·0	5419·5	61·9–62·1	754·0	,,
2709·65 mean	2710·15 mean	5419·8 mean	62·0 mean		
		Scale B.			
st	*tv*	*sv*			
2711·1	2711·9	5423·0	61·9–62·1	754·4	Nil
2710·5	2713·0	5423·5	62·0–62·0	756·0	,,
2710·8 mean	2712·5 mean	5423·3 mean	62·0 mean		
		Scale C.			
βγ	γδ	βδ			
2711·2	2712·0	5423·2	61·8–61·9	761·2	Nil
2710·8	2712·8	5423·6	61·9–62·0	760·3	,,
2711·0 mean	2712·4 mean	5423·4 mean	61·9 mean		

The corrections for temperature and pressure are again too small to affect the decimal of a band.

The results of the counting of the interference bands in the $\frac{1}{16}$-inch intervals on the three scales, A, B, and C, at 62° F., are thus :—

For scale A, *ac* = 5419·8
,, B, *sv* = 5423·3
,, C, βδ = 5423·4.

The differences for the three scales were twice in each case confirmed directly on the comparator, and found to be as shown, 3·5 bands between A (the smaller) and B, and 3·6 between A and C.

The Evaluation of the Graduated Intervals, from the Base-line Results.—The determination of the intervals on the $1\frac{1}{8}$-inch scales was carried out on the comparator in the following manner. To begin, one adjusts the whole invar bar, carrying the fast cemented scales and single location-signals, on the travelling carriage—where it rests on supports at the Airy positions for no flexure of the bar—so that the right-hand microscope (to which the interferometer is attached) is over the end location-signal z of the scale B, and the left-hand microscope over the zero location-signal a of the scale A. When both a and z have been focussed by the high power and their illumination has been adjusted to show the five ruled lines very clearly, the bar must be transversely adjusted so that the two longitudinal finder lines are central in the field, an important adjustment, so that repetitions of determinations can be strictly comparable with each other, and be carried out along the same straight central line for the whole yard-length. With this in view the greatest possible care was taken with the alignment of the rulings in their mounting on the bar, so that the ruled lines were absolutely perpendicular to the length of the bar, and with the longitudinal finder-lines in the central line of the bar-face.

To describe one determination of an interval is practically to describe all, and it will be sufficient to take two cases, say determinations 3 and 7 of the table on p. 307. In the case of 3, we are to find the length of wx, the second $\frac{1}{8}$-inch on scale B (not counting the extra eighth rs), from the known length ad (from operation 1), the first $\frac{1}{8}$ on the scale A. We adjust the left-hand microscope over the zero location-signal a on scale A, and the right-hand microscope over the location-signal w on scale B. Each of these location-signals is to have its central ruling of the five most accurately adjusted between the pair of spider-lines. A good field of interference bands should also be arranged, preferably with a band exactly centred to the silver ring and spider-lines, and then all is ready for the determination. We now gently push the travelling carriage, with the bar immovably resting on it, to the left, along its V-and-plane bed, by means of the fine adjustment fitting detachably provided for the purpose at the right end of the carriage, until the location-signal d on scale A is accurately adjusted under the left-hand microscope. If one goes too far, as often happens, one can come back with the aid of the similar fitting at the left end of the carriage, releasing the right fitting while using the left one, or *vice versa*. If wx is exactly equal to ad the location-signal at x on scale B should now be found precisely adjusted under the right-hand microscope. But even Professor GRAYSON's great skill could not attain such perfection to so delicate a test, and wx proved a little longer than ad, the five rulings of x being slightly to the left of the spider-lines (the microscope inverting). The big wheel of the interferometer was then rotated slowly in the correct direction to bring the microscope exactly to adjustment over x, the middle line of the five precisely between the spider-lines, counting the interference bands which passed in the process. On the three different days on which the observations were made the numbers of bands were respectively 6·5, 8·0, and 7·2. These added to the known length of ad gave the true length of wx.

The case of operation 7 was similarly dealt with. The second $\frac{1}{2}$-inch interval on A, fg, was to be determined, having given, from the result of operation 6, the $\frac{1}{2}$-inch interval sy on B. The left-hand microscope was placed over f, and the right-hand one over s, and both location-signals were adjusted. The carriage was then pushed over to the left, partly by hand as the interval was relatively long for the fine adjustment limits, and partly by the right fine adjustment fitting, until the left microscope was over the location-signal *g* on scale A, the 1-inch mark. After the middle line of the five of this signal had been precisely adjusted, one moved over to the right microscope, to find that the signal *y* of scale B was nearly but not quite adjusted to the spider-lines of that microscope, *fg* being slightly longer than *sy*, by an amount which on the three different days corresponded to the passage of 7·5, 5·0, and 5·0 bands respectively. On adding these to the band-content of *sy*, the value of *fg* was at once afforded.

The evaluation of the longer intervals defined by the single location-signal rulings is very similar, up to a point. For instance, operation 12, the measurement of *pq*, the second $2\frac{1}{4}$-inch interval from the B end, may be considered, having given *ak*, the first $2\frac{1}{4}$-inch interval on A. Placing the left-hand microscope over *a* and the right-hand one over *p*, and adjusting both location-signals to the spider-lines, we push the bar-carriage to the left by hand, until it is nearly in position, then completing the movement with the right-hand fine adjustment fitting, until *k* is under the left microscope, when that location-signal can be delicately adjusted between the spider-lines. The right-hand microscope should then be over the ruling *q*. On looking into that microscope, the ruling was seen to be almost exactly adjusted to the spider-lines, being arranged so that the spider-lines were just on or slightly within the first ruling of the five, *pq* being infinitesimally shorter than *ak*, obviously about or just under four band-equivalents. On using the interferometer, the exact amount of the difference was indeed found on the three different days to be 4·0, 3·1, and 3·5 bands.

So far the beam-compass method of using the two comparator microscopes has not been available, as the microscope-fittings do not admit of the microscopes being approached nearer to each other than 4 inches. But it begins to be available with the $4\frac{1}{2}$-inch intervals, and if we take number 13 we shall have an example of it. The problem is to find the length of *op*, the second $4\frac{1}{2}$-inch interval from the B end, having given *pz*, the first $4\frac{1}{2}$-inch interval from B. Arranging that the location-signal *z* is adjusted under the right-hand microscope, we bring the left-hand microscope to the right, moving its supporting block (the lower main one) by hand over the main V-and-plane bed, and fine-adjusting the upper block until the microscope is over the location-signal *p*. We then push the bar-carriage to the right till the left microscope is over location-signal *o*. The right microscope should then be over the signal *p*. On looking into that microscope one saw the location-signal somewhat to the left of the spider-line, indicating that *op* was longer than *pz*. On rotating the big wheel of the interferometer, to bring the middle ruling to the centre of the spider-lines, counting bands the while, one found on the three different days that 24·5, 26·3 and 24·6 bands passed the reference centre respectively.

The last of these beam-compass operations, and the last of all the measurements on the author's yard, is the determination of the second half yard, operation 15. It is conducted in the same way, except that after the location signals n (at 18 inches) and z (at 36 inches) have been adjusted under the left and right microscopes respectively, the bar itself is taken up off the carriage, turned round, and laid down again on the carriage, but so that while n comes again under the left microscope it is now a which comes under the right microscope. Before we can finally adjust the two signals n and a, it is necessary again to adjust the longitudinal finder lines to the central line of measurement, as this adjustment, of course, was destroyed by the inversion of the bar. When satisfied that the location-signal n is adjusted to the spider-lines of the left microscope, while the longitudinal finders are also central in the field, we observe the situation of a under the right-hand microscope. The location-signal was seen to be well in the field, but rather further from the centre than for other rulings, and it took on the 3 days 43·5, 42·6 and 41·6 bands of movement to centre the middle ruling between the spider-lines. On adding these numbers to the already known band-value of nz, the number in the other half-yard an was obtained; then adding an and nz together, the number of interference bands in red hydrogen light (H_α) corresponding to the author's yard az was arrived at.

The Corrections for Temperature and Pressure.—It is well known that the absolute wave-length of a light radiation in air varies slightly with the refractive index of air, which in turn is variable with change of atmospheric conditions, density, temperature, pressure, and humidity. Following a reasoning concordant with FABRY and PEROT (*loc. cit.*, p. 43), and with A. PERARD,* if λ_a be the wave-length under the actual observed conditions of pressure H, temperature θ, and vapour pressure f, and λ_n be that for the standard conditions of 15° C. and 760 mm.; and if N be the refractive index of air under the observed conditions and N_n that for the standard conditions, then

$$\frac{\lambda_a}{\lambda_n} = \frac{N_n}{N} = 1 + A - B\,\frac{H}{1+\alpha\theta} + Cf,$$

where

$$A = \frac{n_0 - 1}{n_0 + 15\alpha}, \quad \text{and} \quad B = \frac{(n_0 - 1)(1 + 15\alpha)}{(n_0 + 15\alpha)\,760};$$

in which n_0 is the refractive index of air at 0° C. and 760 mm., and α the coefficient of expansion of air. The term Cf may be neglected, for any appreciable effect is compensated by the slight thermal expansion of the invar bar, the coefficient of which is given on p. 297.†

FABRY and PEROT give a simple couple of rules for arriving at the corrections, in agreement with the above formulæ. They are as follows:—

The order of interference (number of bands) increases by $3\cdot65 \times 10^{-6}$ of its value for 1 cm. increase of pressure. And it diminishes by $0\cdot96 \times 10^{-6}$ of its value for 1° C.

* 'Trav. et Mém. Bur. International des Poids et Mesures,' vol. 18, p. 43 (1929).

† The temperature effect on the index of air is $1\cdot07 \times 10^{-6}$, and the coefficient of expansion of the invar bar $1\cdot12 \times 10^{-6}$. The slight difference allows for the humidity effect.

Evaluation of the $1\frac{1}{8}$-inch Grayson-Ruling Scales A, B, C, from the counted Basal $\frac{1}{16}$-inch Intervals, in Red Hydrogen (H_α) Light.

Interval to determine.	Given.	Observation on comparison.	By how many interference bands. I.	II.	III.	Result.	Further consequence of result.
	ac = 4832·7						
1. cd (2nd $\frac{1}{16}$-inch on A)	sv = 4835·8	cd shorter than sv	0·4			4835·4	ad (the $\frac{1}{8}$-inch on A) = $ac + cd$ = 9668·1
		,,		0·2		4835·6	,, ,, = 9668·3
	$\beta\delta$ = 4835·8	cd longer than $\beta\delta$			0·5	4836·3	,, ,, = 9669·0
2. sw ($\frac{1}{8}$-inch on B)	ad = 9668·1	sw longer than ad	0·2			9668·3	
	9668·3	,,		1·2		9669·5	
	9669·0	,,			0·7	9669·7	
3. wx (2nd $\frac{1}{8}$-inch on B)	ad = 9668·1	wx longer than ad	6·5			9674·6	sx (the $\frac{1}{4}$-inch on B) = $sw + wx$ = 19342·9
	9668·3	,,		8·0		9676·3	,, ,, = 19345·8
	9669·0	,,			7·2	9676·2	,, ,, = 19345·9
4. ae ($\frac{1}{4}$-inch on A)	sx = 19342·9	ae shorter than sx	2·2			19340·7	
	19345·8	,,		3·0		19342·8	
	19345·9	,,			3·0	19342·9	
5. ef (2nd $\frac{1}{4}$-inch on A)	sx = 19342·9	ef longer than sx	7·8			19350·7	af (the $\frac{1}{2}$-inch on A) = $ae + ef$ = 38691·4
	19345·8	,,		4·7		19350·5	,, ,, = 38693·3
	19345·9	,,			4·7	19350·6	,, ,, = 38693·5
6. sy ($\frac{1}{2}$-inch on B)	af = 38691·4	sy shorter than af	1·2			38690·2	
	38693·3	sy longer than af		0·6		38693·9	
	38693·5	sy shorter than af			1·2	38692·3	
7. fg (2nd $\frac{1}{2}$-inch on A)	sy = 38690·2	fg longer than sy	7·5			38697·7	ag (the 1-inch on A) = $af + fg$ = 77389·1
	38693·9	,,		5·0		38698·9	,, ,, = 77392·2
	38692·3	,,			5·0	38697·3	,, ,, = 77390·8
8. gh (extra $\frac{1}{8}$-inch on A)	wx = 9674·6	gh shorter than wx	0·4			9674·2	ah (the $1\frac{1}{8}$-inch on A) = $ag + gh$ = 87063·3
	9676·3	,,		1·4		9674·9	,, ,, = 87067·1
	9676·2	,,			2·9	9673·3	,, ,, = 87064·1
9. rz ($1\frac{1}{8}$-inch on B)	ah = 87063·3	rz longer than ah	12·5			87075·8	
	87067·1	,,		12·3		87079·4	
	87064·1	,,			12·5	87076·6	

of rise of temperature. These rules are most useful, and have been employed in calculating the corrections to the number of bands actually observed.

As the intervals on the $1\frac{1}{8}$-inch scales were all always observed on the same day, generally within 3 hours, the corrections were applied to the total number of bands found for the full $1\frac{1}{8}$-inch, the temperatures and pressures having been observed before and after the measurements, and the mean employed; for the limits were practically never more than 0·2° F. from the official temperature 62° F. (16·667° C.). Similarly the longer intervals formed by the single location-signals were also all determined on the same day, within 3 hours, and the temperatures and pressures observed before and after, and the mean used. Several times the whole set of measurements (the complete yard) were carried out on the same day, one in the morning and the other in the afternoon, but they were treated for correction purposes as just described, namely, as separate operations. As regards the temperatures not only were the three standardised thermometers at suitable positions near the comparator or interferometer read, but a fourth one actually in contact with the instrument, near the bar or ruling, was also observed; in the case of the comparator it was laid in the groove of the reference standard bar of 1926 alongside the invar bar. No appreciable effect beyond 0·1°–0·2° was ever observed with it, due to the observer's body proximity, which was only intermittent and avoided when not actually observing.

The lengths in red hydrogen interference bands, of the two $1\frac{1}{8}$-inch Grayson-ruling scales A and B, given at the close of the preceding table, as the results of the evaluation operations, carried out on three different days (I, II, and III), are uncorrected for the temperature and pressure during those operations. In the following summary table the corrections are given and applied.

	Uncorrected number of bands.		Net correction for temperature and pressure.	Corrected number of bands.	
	ah (on A).	*rz* (on B).		*ah.*	*rz.*
I	87063·3	87075·8	—0·2	87063·1	87075·6
II	87067·1	87079·4	—0·1	87067·0	87079·3
III	87064·1	87076·6	+0·1	87064·2	87076·7
			Mean corrected values .	87064·8	87077·2

The above net corrections were obtained as follows :—

The barometric readings on the days I, II and III were 763·8, 767·1 and 756·0 mm. respectively, and the temperatures were 62·1°, 61·9° and 62·0° F.

For the pressure we have (using Fabry and Perot's formula-rule) :—

Subtract $0\cdot000\ 003\ 65 \times 87070 = 0\cdot3178$ for 10 mm. increase of pressure,

and $0\cdot3178 \times 0\cdot38 = 0\cdot1$ for I

$0\cdot3178 \times 0\cdot71 = 0\cdot2$ for II

$0\cdot3178 \times 0\cdot40 = 0\cdot1$ for III.

The pressure corrections are therefore $-0\cdot1$, $-0\cdot2$ and $+0\cdot1$ respectively.

The temperature corrections are of two opposing kinds, one for change of density of the air and therefore of wave-length of the light, and one for the thermal expansion of the speculum metal slabs on which Professor GRAYSON ruled his location-signal-lines. The coefficient of expansion of the speculum metal was 0·000 019 33 per 1° C. Per degree Fahrenheit we multiply this by five-ninths, and then the length (in bands) expanded by the interval corresponding to 87070 interference bands will be:

$$\frac{0\cdot000\ 019\ 33 \times 5 \times 87070}{9} = 0\cdot94 \text{ bands per } 1° \text{ F.}$$

For the temperatures 62·1°, 61·9° and 62·0° F., the differences from the standard temperature were only 0·1°, 0·1° and nil during the three operations I, II and III respectively. The corrections on this account are therefore − 0·09, +0·09, 0·0 bands respectively, the signs of the corrections being as shown.

The corrections on account of alteration of wave-length, we have from FABRY and PEROT's formula-rule that the number of bands (order of interference) diminishes $0\cdot96 \times 10^{-6}$ of their value for 1° C. rise of temperature. For 1° F. this becomes $0\cdot96 \times \frac{5}{9}$ or 0·000 000 53 per 1° F. For a length corresponding to 87070 bands:

$$87070 \times 0\cdot000\ 000\ 53 = 0\cdot046 \text{ per } 1° \text{ F. to be added.}$$

This is obviously negligible, not affecting the first decimal place, as the differences from 62° F. are only one-tenth of a degree.

Thus the corrections summarise as follows, and are those used in the table on p. 308 in correcting the observed numbers of interference bands:—

	For pressure.	For temperature.	Net correction.
I	−0·1	−0·1	−0·2
II	−0·2	+0·1	−0·1
III	+0·1	Nil	+0·1

Referring now to the next table (p. 310), the mean temperature for determination I was 16·69° C. (limits 16·57°–16·80°); for II it was 16·67° (16·62°–16·72°); and for III it was 16·66° (16·62°–16·69°), all remarkably close to the standard temperature of 62° F. (16·67° C.). But no corrections for temperature are required within such close limits; for the extremely slight expansion or contraction of the invar bar on which the GRAYSON signal-rulings are mounted so nearly absolutely balances the contrary effect of change of temperature on the refractive index of air and also the condition of air-humidity, on the wave-length of the light radiation used to produce the interference bands, that the net difference was utterly negligible.

The pressure corrections to 760 mm. are appreciable. On the three days I, II, III the barometric readings were 752·3 mm., 763·7 mm., and 759·4 mm. respectively, the differences from 760 mm. being thus −7·7, +3·7, and −0·6 mm. The correction is

Evaluation of the Longer Intervals marked by Single GRAYSON Location-signals, and of the Yard, in Red Hydrogen Light (H_{α}).

Interval to determine.	Given.	Observation on comparison.	By how many interference bands. I.	II.	III.	Result.	Further consequence of result.
10. *hk* (2nd $1\frac{1}{8}$-inch from A end)	*ah* = 87064·8	*hk* longer than *rz*	17·7			87094·9	*ak* ($2\frac{1}{4}$ inches from A end)=*ah*+*hk*=174159·7
	rz = 87077·2			16·3		87093·5	,, ,, =174158·3
					17·4	87094·6	,, ,, =174159·4
11. *qz* ($2\frac{1}{4}$ inches from B end) .	*ak* = 174159·7	*qz* shorter than *ak*	6·3			174153·4	
	174158·3			5·2		174153·1	
	174159·4				6·0	174153·4	
12. *pq* (2nd $2\frac{1}{4}$-inches from B end)	*ak* = 174159·7	*pq* shorter than *ak*	4·0			174155·7	*pz* ($4\frac{1}{2}$ inches from B end)=*qz*+*pq*=348309·1
	174158·3			3·1		174155·2	,, ,, =348308·3
	174159·4				3·5	174155·9	,, ,, =348309·3
13. *op* (2nd $4\frac{1}{2}$-inches from B end)	*pz* = 348309·1	*op* longer than *pz*	24·5			348333·6	*oz* (9 inches from B end)=*pz*+*op*=696642·7
	348308·3			26·3		348334·6	,, ,, =696642·9
	348309·3				24·6	348333·9	,, ,, =696643·2
14. *no* (2nd 9-inches from B end)	*oz* = 696642·7	*no* shorter than *oz*	11·5			696631·2	*nz* (18 inches from B end)=*oz*+*no*=1393273·9
	696642·9			9·1		696633·8	,, ,, =1393276·7
	696643·2				11·0	696632·2	,, ,, =1393275·4
15. *an* (18 inches from A end) .	*nz* = 1393273·9	*an* longer than *nz*	43·5			1393317·4	*az* (the yard)=*nz*+*an*=2786591·3
	1393276·7			42·6		1393319·3	,, ,, =2786596·0
	1393275·4				41·6	1393317·0	,, ,, =2786592·4

(as already explained in the introduction) 2786593 × 0·000 003 65 = 10·17 interference bands, for 10 mm. change of pressure, or 1·017 for 1 mm.

For the three determinations the pressure corrections are therefore :—

7·7 × 1·017 = 7·8 bands for determination		I.
3·7 × 1·017 = 3·8	,,	II.
0·6 × 1·017 = 0·6	,,	III.

The signs will be +, —, and +, as the correction has to be subtracted for increase of pressure above 760 mm. (as the order of interference, number of bands, is increased by increase of pressure, and is thus greater than it should be), and added for diminution of pressure below 760 mm. Applying these corrections we have, for the number of interference bands in red hydrogen light (H_α) in the author's yard, *az* :—

For determination	I	2786591·3 + 7·8 = 2786599·1
,,	II	2786596·0 — 3·8 = 2786592·2
,,	III	2786592·4 + 0·6 = 2786593·0

Comparison of the Author's Yard, az, with the 1926 *Standard Yard.*

Over 40 comparisons have been made of this yard, *az*, with the standard yard of 1926, at temperatures of 62·0° F. or within 0·1° of that standard temperature. The exact length of the 1926 yard, as compared, with the greatest attainable accuracy, with the Imperial Standard Yard prototype, is 36·000443 inches at 62° F. The comparisons with *az* concur, within a single band, in showing that the length *az* is shorter than the 1926 standard yard by (in the mean) 20·5 interference bands in red hydrogen (H_α) light for exactly 62° F.

The 1926 standard yard's fraction of an inch, 0·000443, by which it is longer than the prototype yard, corresponds to 0·000443 × 77389 = 34·28 bands. For the author's inch (No. 7 determination on p. 307) has been shown to correspond to 77389 interference bands in red H_α light. Thus the author's yard *az* is of intermediate length between the Imperial prototype and 1926 yards.

Hence, in order to deduce from the author's yard *az* the number of H_α interference bands corresponding to the Imperial Standard Yard, 13·8 bands require to be subtracted from the results given above on this page. For 34·3 — 20·5 = 13·8, that is. *az* is longer than the prototype by 13·8 bands. Making this deduction we get, for the total number of H_α interference bands in the prototype :—

From determination	I	2786599·1 — 13·8 = 2786585·3
,,	II	2786592·2 — 13·8 = 2786578·4
,,	III	2786593·0 — 13·8 = 2786579·2

As the interference bands correspond to half wave-lengths we have finally :—

Wave-lengths of Red Hydrogen Light (H_a) *in the Imperial Standard Yard.*

From determination	I	1393292·7	mean **1393290·5**
,,	II	1393289·2	
,,	III	1393289·6	

The Content of Cadmium-red Wave-lengths in the Yard.

To calculate now the number of wave-lengths of the red light of cadmium, from this number found for red hydrogen light and the two wave-lengths of Cd_r and H_a, we have :

$$\frac{1393290 \cdot 5 \times 6562 \cdot 8473}{6438 \cdot 4698} = \mathbf{1420205 \cdot 9} \text{ wave-lengths } Cd_r.$$

The value used for the wave-length of H_a, 6562·8473, is that due to HOUSTON (see p. 319 for discussion of the wave-length of H_a), for the brighter component of the close doublet, which is shown on p. 320 to be very probably the effective radiant. If, however, the optical centre of gravity of the doublet be used instead, 6562·791, or the almost identical value of CURTIS, 6562·793 for the unresolved line, the number of Cd_r wave-lengths would be 1420194.

Determinations with Neon-yellow Light.

The measurements with neon light will now be given. An excellent vacuum tube of the GUILD pattern (H-shape with large bulb blown on one of the limbs) was used, and the rectangular aperture in front of the small totally reflecting prism at the auto-collimating centre of the interferometer-telescope was narrowed to almost a slit, so as to exclude other yellow-line-rays than those of the desired Ne_c. The iris diaphragm of the whole circular aperture at this common focal centre was also rendered as small as possible, in order to exclude all other radiations. The interference bands were still adequately brilliant, as the light of Ne_c is so overpoweringly intense compared with that from the other yellow lines, and the bands themselves were very black on the bright yellow ground. If this be not done the black bands are apt to become somewhat illuminated periodically by secondary interference from one or other of these other lines ; yet even then the black due to Ne_c predominates sufficiently for accurate counting. By reducing the rectangular aperture to the width of a broad slit, however, this drawback is almost entirely avoided.

The results for the measurements of the $1\frac{1}{8}$-inch ruled scales, and of the longer intervals formed by the single location-signal rulings, are given in the next two long tables, the necessary corrections for atmospheric conditions, and the results of their application, being appended to each in a subsidiary table. The C-ruling was used to confirm the A and B values.

Evaluation of the $1\frac{1}{8}$-inch GRAYSON-ruling Scales A, B, C from the Counted Basal $\frac{1}{16}$-inch Intervals in Yellow Neon Light.

Interval to determine.	Given.	Observation on comparison.	By how many interference bands. I.	II.	III.	Result.	Further consequence of result.
1. *cd* (2nd $\frac{1}{16}$-inch on A)	*ac* = 5419·8	*cd* greater than *sv*	0·7			5424·0	*ad* (the $\frac{1}{8}$-inch on A)=*ac*+*cd*=10843·8
	sv = 5423·3	*cd* less than *sv*		0·2		5423·1	„ „ =10842·9
		cd equal to *sv*			—	5423·3	„ „ =10843·1
2. *sw* ($\frac{1}{8}$-inch on B)	*ad* = 10843·8	*sw* greater than *ad*	0·3			10844·1	
	10842·9	„		0·3		10843·2	
	10843·1	„			0·5	10843·6	
3. *wx* (2nd $\frac{1}{8}$-inch on B)	*ad* = 10843·8	*wx* greater than *ad*	7·2			10851·0	*sx* (the $\frac{1}{4}$-inch on B)=*sw*+*wx*=21695·1
	10842·9	„		8·4		10851·3	„ „ =21694·5
	10843·1	„			8·5	10851·6	„ „ =21695·2
4. *ae* ($\frac{1}{4}$-inch on A)	*sx* = 21695·1	*ae* less than *sx*	2·5			21692·6	
	21694·5	„		3·3		21691·2	
	21695·2	„			4·0	21691·0	
5. *ef* (2nd $\frac{1}{4}$-inch on A)	*sx* = 21695·1	*ef* greater than *sx*	7·4			21702·5	*af* ($\frac{1}{2}$-inch on A)=*ae*+*ef*=43395·1
	21694·5	„		8·0		21702·5	„ „ =43393·7
	21695·2	„			7·0	21702·2	„ „ =43393·2
6. *sy* ($\frac{1}{2}$-inch on B)	*af* = 43395·1	*sy* less than *af*	1·2			43393·9	
	43393·7	„		0·6		43393·1	
	43393·2	„			0·6	43392·6	
7. *fg* (2nd $\frac{1}{2}$-inch on A)	*sy* = 43393·9	*fg* greater than *sy*	6·0			43399·9	*ag* (the 1-inch on A)=*af*+*fg*=86795·0
	43393·1	„		7·2		43400·3	„ „ =86794·0
	43392·6	„			7·2	43399·8	„ „ =86793·0
8. *gh* (extra $\frac{1}{8}$-inch on A)	*wx* = 10851·0	*gh* less than *wx*	0·5			10850·5	*ah* (the $1\frac{1}{8}$-inch on A)=*ag*+*gh*=97645·5
	10851·3	„		1·4		10849·9	„ „ =97643·9
	10851·6	„			1·4	10850·2	„ „ =97643·2
9. *rz* ($1\frac{1}{8}$-inch on B)	*ah* = 97645·5	*rz* greater than *ah*	10·0			97655·5	
	97643·9	„		9·9		97653·8	
	97643·2	„			10·1	97653·3	

To give greater variety and afford independent mutual confirmation of the final results to be given later, the operations from 12 to 15 inclusive have been performed on similar intervals on different (the right and left) halves of the bar, one side being used for the determinations in hydrogen light, and the other side for the operations in neon light. Thus, while in No. 13 the right-hand interval *op* has been determined in hydrogen light, the corresponding interval on the left *lm* has been measured in neon light. This variation has been possible by reason of the symmetrical arrangement of the single location-signal rulings, two more of the single rulings (either *l* and *m* or *p* and *o*) having been mounted than were absolutely necessary. Also it should be recorded that the central ruling at 18 inches was remounted between the hydrogen and neon operations, accidental contact with one of the objectives rendering this desirable.

The corrections to be applied are as follows. The barometric pressures on the 3 days of the determinations were 767·1, 760·7 and 762·3 respectively, and the mean temperatures 62·0°, 61·9° and 61·8° F., the limits of the temperatures throughout the whole of the observations having never exceeded 0·2° F. from 62·0°.

For the pressure corrections we have: $0{\cdot}00000365 \times 97650 = 0{\cdot}3564$ band for 10 mm. of pressure increase, and 0·3564 multiplied respectively by 0·71, 0·07 and 0·23 give 0·26, 0·02 and 0·08, or 0·3, nil, and 0·1, the signs being negative. For the temperature corrections, that due to alteration of wave-length is: $0{\cdot}000\,000\,53 \times 97650 = 0{\cdot}0527$ band per 1° F. As the temperatures never differed more than a fifth of a degree Fahrenheit, this part of the correction is obviously negligible. That due to thermal dilatation of the little speculum metal slab on which the rulings are engraved is:

$$\text{Five-ninths of } 0{\cdot}000\ 019\ 33 \times 97650 = 1{\cdot}05 \text{ band per } 1^\circ \text{ F.}$$

As the temperature differences were nil, —0·1 and —0·2, the corrections are 0, +0·1 and +0·2. These several corrections are now applied below.

	Uncorrected number of bands.		Corrections— For *p*.	For *t*.	Net total	Corrected number of bands	
	ah	*rz*				*ah*	*rz*
I	97645·5	97655·5	—0·3	Nil	—0·3	97645·2	97655·2
II	97643·9	97653·8	Nil	+0·1	+0·1	97644·0	97653·9
III	97643·2	97653·3	—0·1	+0·2	+0·1	97643·3	97653·4
Mean						97644·2	97654·2

As regards the next table (p. 315), the mean temperatures, during the three observations on different days I, II and III were 16·70°, 16·66° and 16·66° C. respectively, the extreme limits being never more than 0·06° C. from the standard temperature of 16·667° C. (62·0° F.). The corrections on account of temperature are, however, again *nil*, the minute thermal expansion of the invar bar and the opposite effect of change of temperature on the wave-length absolutely balancing each other within 0·1 band.

Evaluation of the Longer Intervals, including the Yard, in Yellow Neon Light.

Interval to determine.	Given.	Observation on comparison.	By how many bands. I.	II.	III.	Result.	Further consequence of result.
	ah = 97644·2						
10. *hk* (2nd $\frac{1}{8}$-inch at A end)	*rz* = 97654·2	*hk* greater than *rz*	25·3			97679·5	*ak* ($2\frac{1}{4}$ inches from A end)=*ah*+*hk*=195323·7
		,,		25·3		97679·5	,, ,, =195323·7
		,,			26·5	97680·7	,, ,, =195324·9
11. *qz* ($2\frac{1}{4}$ inches from B end)	*ak* = 195323·7	*qz* less than *ak*	28·2			195295·5	
	195323·7	,,		29·0		195294·7	
	195324·9	,,			29·5	195295·4	
12. *kl* (2nd $2\frac{1}{4}$ inches from A end)	*qz* = 195295·5	*kl* greater than *qz*	14·3			195309·8	*al* ($4\frac{1}{2}$ inches from A end)=*ak*+*kl*=390633·5
	195294·7	,,		14·6		195309·3	,, ,, =390633·0
	195295·4	,,			14·5	195309·9	,, ,, =390634·8
13. *lm* (2nd $4\frac{1}{2}$ inches from A end)	*al* = 390633·5	*lm* less than *al*	28·2			390605·3	*am* (9 inches from A end)=*al*+*lm*=781238·8
	390633·0	,,		28·0		390605·0	,, ,, =781238·0
	390634·8	,,			31·0	390603·8	,, ,, =781238·6
14. *mn* (2nd 9 inches from A end)	*am* = 781238·8	*mn* less than *am*	28·5			781210·3	*an* (18 inches from A end)=*am*+*mn*=1562449·1
	781238·0	,,		27·0		781211·0	,, ,, =1562449·0
	781238·6	,,			27·5	781211·1	,, ,, =1562449·7
15. *nz* (18 inches from B end)	*an* = 1562449·1	*nz* less than *an*	66·0			1562383·1	*az* (the yard)=*an*+*nz*=3124832·2
	1562449·0	,,		64·0		1562385·0	,, ,, =3124834·0
	1562449·7	,,			66·1	1562383·6	,, ,, =3124833·3

The pressures during the measurements were for I 761·0 mm., for II 760·7 mm., and for III 762·3 mm. The correction is 3124833 × 0·000 003 65 = 11·4 bands, for 10 mm. pressure charge. The pressure differences from 760·0 mm. being +1·0, +0·7 and +2·3, the corrections are —1·1 band, —0·8 band, and —2·6 bands for the three days I, II, III.

Applying these corrections we have, for the number of interference bands in yellow neon light contained in the author's yard, *az* :—

For determination	I . . .	3124832·2 — 1·1 = 3124831·1	Mean 3124831·7
,,	II . .	3124834·0 — 0·8 = 3124833·2	
,,	III . .	3124833·3 — 2·6 = 3124830·7	

The results are thus even more satisfactorily concordant than those for red hydrogen light.

Comparison of the Author's Yard, az, with the 1926 *Standard Yard.*

As for the determinations in H_α light, so for neon yellow light a large number of direct comparisons were made of the author's yard with the 1926 standard bar. The excess of this latter, of 0·000443 inch, over the Imperial Standard Yard, corresponds to 38·5 bands in neon light. For 0·000443 × 86794 (the bands in 1 inch) = 38·5. The mean of all the comparisons of the two yards (*az* and the 1926 bar), for the temperature 62·0° F. (the limits never exceeding 0·01 during these comparisons), indicated that the author's bar *az* is shorter than the 1926 standard by 24·0 interference bands in neon light, the extreme limits of the differences in the separate observations being never more than half a band on each side of this number.

The author's yard *az* is therefore 38·5 — 24·0 = 14·5 bands longer than the Imperial Standard Yard, that is, the latter comprises

$$3124831 \cdot 7 - 14 \cdot 5 = 3124817 \cdot 2 \text{ bands.}$$

Dividing this by two, as the interference bands correspond to half wave-lengths, we have for the number of wave-lengths of yellow neon light comprised at 62° F. and 760 mm. pressure, in the Imperial Standard Yard

1562408·6.

The Content of Cadmium-red Wave-lengths in the Yard.

We are now in a position to calculate the number of wave-lengths of the red radiation of cadmium contained in the Imperial Standard Yard, from this value for neon yellow light and the wave-lengths of the two radiations. For the wave-length of this brilliant neon line Ne_c is thoroughly well known (see p. 318), 5852·488 Å.U., as determined interferometrically with direct reference to the standard wave-length of the cadmium red line Cd_r fixed by MICHELSON and by BENOIT, FABRY and PEROT at 6438·4698 Å.U. The simple calculation is, therefore

$$\frac{1562408 \cdot 6 \times 5852 \cdot 488}{6438 \cdot 4698} = \mathbf{1420209 \cdot 8} \text{ wave-lengths of } Cd_r.$$

It will be interesting to consider how this result for the number of Cd_r wave-lengths in the Imperial Standard Yard accords with a result obtained in the year 1927 for the relation between the yard and the metre, by Messrs. J. E. SEARS (Deputy Warden of the Standards), W. H. JOHNSON, and H. P. L. JOLLY,* employing the most accurate non-interferential methods, partly at the National Physical Laboratory, partly at the Bureau International des Poids et Mesures at Sèvres, and partly at the Standards Department of the Board of Trade, employing the mechanical, microscopic and micrometric portions of the "Tutton comparator."† They found that

$$1 \text{ yard} = 0 \cdot 91439841 \text{ metre}.$$

Now the number of wave-lengths of cadmium red light found in the metre by MICHELSON and by BENOIT, FABRY and PEROT, were :—

MICHELSON	1553163·50	Mean 1553163·82.
BENOIT, FABRY and PEROT	1553164·13	

Hence, combining the results :—

$$1553163 \cdot 82 \times 0 \cdot 91439841 = 1420210 \cdot 53.$$

The close agreement between this result and that of the author 1420209·8, from the perfectly unambiguous determination with neon light, cannot but be regarded as satisfactory. It may be mere chance that has made the concordance so very close, yet there can be little doubt that the round number 1420210 is very near the truth. For although the differences between the individual determinations amount in the case of the hydrogen set to 3·5 wave-lengths, and in the neon set to 1·3 wave-lengths, the mean value has in each case every probability of being nearer the truth than any one of the individual values. For all the possible errors are those involved in the allocation of a location-signal ruling (including its focussing by the very high-power objective) or an interference band to the spider-lines or reference centre, and thus have equal chances of being on either side of absolute accuracy.

The slight differences between the interferometric corrections for the same interval on different days may be due in part to slight movement of the little speculum metal slabs on the invar bar during the intervening non-observing days. During the two hours of actual observation no movement was ever detected. The final results for the different

* 'Phil. Trans.,' A, vol. 227, p. 281 (1928).

† '*Loc. cit.*, A, vol. 210, p. 1 (1909).

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

days thus agree more perfectly than might be expected. In the case of the intervals on the $1\frac{1}{8}$-inch scales, the slight differences on different days are due mainly to the focussing variations with so high a power.

The author desires to make it clear, however, that he does not claim the extreme accuracy which the final numerical results of this determination show. As regards any possible error in the counting of the interference bands in the $\frac{1}{8}$-inch base-line, which would be repeated 8×36 times in the yard, the count was carried out so often, and on three sets of rulings, that any error in the final mean could not possibly exceed 0·03 of an interference band, and is probably less. This would multiply up to $8\frac{1}{2}$ bands, or $4\frac{1}{4}$ wave-lengths, at the maximum, and the " probable error " would be less than this; the author is convinced that the real error is considerably less than this. For not only were more counts made than are recorded, all within the limits of those records, but also the checks and counterchecks by comparisons between the three sets of rulings were so numerous that no doubt on this score was possible. The observations for yellow neon light are those in which the author has the greatest confidence. For the calculation of the cadmium red content of the yard from the yellow neon result is absolutely free from ambiguity, as the wave-lengths of both radiations are now standard constants, interferometrically determined, while the hydrogen result is not so free from doubt on this score, a matter which must next be discussed.

The Wave-length of the Red Hydrogen Line H_α, *and its Bearing on the Results.*

With regard to the wave-lengths used in this investigation, that taken for the universal standard radiation, the red cadmium line Cd_r in air at the temperature of 15° C. and 760 mm. barometric pressure, is the mean of the value of MICHELSON,* as corrected by BENOIT, FABRY and PEROT, and that of the last mentioned authors. MICHELSON himself gave 6438·4722 Å.U. To this was subsequently applied an omitted correction (Benoit, Fabry, and Perot†), bringing it to 6438·4700. The value obtained by BENOIT, FABRY and PEROT as the result of their own work‡ was 6438·4696. The mean of these two last stated values is 6438·4698, for both researches are equally excellent and trustworthy, and it is this mean value which is employed by the author. With respect to neon, an excellent interferometric determination was made in 1918 by K. BURNS, W. F. MEGGERS and P. W. MERRILL,§ of the wave-length of the intense yellow line Ne_c at 15° C. and 760 mm., the value found being 5852·488 Å.U. This value, now acknowledged as a secondary standard, is used by the author. A confirmatory value (5852·487 – 5852·489) has since (1927) been obtained, also interferometrically, by A. PÉRARD.|| These values for cadmium red and neon yellow are thus

* *Loc. cit.* (' Trav. et Mém.,' 1895), p. 85.

† *Loc. cit.* (' Trav. et Mém.,' 1913), p. 133, and ' Comptes Rendus.,' vol. 144, p. 1082 (1907).

‡ *Loc. cit.* (' Trav. et Mém.,' 1913), p. 131.

§ ' Bull. Bur. Standards,' vol. 14, p. 765 (1918).

|| ' Comptes Rendus,' vol. 184, p. 447 (1927).

absolutely comparable with each other, for the same standard conditions, and the numbers of wave-lengths of these two radiations in the British yard, as now given by the author, are thus particularly trustworthy on this account.

As regards H_α, W. E. CURTIS, working in Professor A. FOWLER'S laboratory in 1914,* found for the unresolved H_α line 6562·793. More recently W. V. HOUSTON,† using a FABRY and PEROT interferometer, found 6562·8473 for the wave-length of the much intenser line, and 6562·7110 for the weaker line, of the doublet, which T. R. MERTON‡ in 1920 had shown, by means of an echelon grating, the line H_α really to be, the two sharp lines being stated to be 0·145 Å.U. apart. The "optical centre of gravity" was about 6562·791, very close to the value of CURTIS, taking into consideration the relative intensities 10 : 7 of the two component lines as previously given by MERTON and NICHOLSON in 1917.§ HOUSTON used the helium line 5015·675 as standard, and describes his result as probably too low according to the standard of Cd_r, as the other helium lines were all also low.

We can obtain a value for the wave-length of H_α by calculating the number of wave-lengths of H_α in the metre, from the number in the yard, using the relation-factor 0·91439841 of SEARS, JOHNSON and JOLLY (see p. 317) as having been shown on p. 317 to agree with the neon result of this research. If x be the desired wave-length of H_α,

$$x = \frac{1 \text{ metre}}{\text{Number of } H_\alpha \text{ wave-lengths in metre}}.$$

For $x \times$ Number of wave-lengths in metre = 1 metre.

Hence

$$x = \frac{1000 \text{ mm.}}{\dfrac{1393290\cdot5 \text{ wave-lengths in yard}}{0\cdot91439841}} = 6562\cdot869 \text{ Å.U.}$$

This new value for the wave-length of H_α is, as expected from the foregoing considerations, slightly higher than that of HOUSTON.

The fact of the red hydrogen line H_α being a doublet (only separable by the phenomenally high resolution of the echelon spectroscope) does not at all interfere with its use in this present determination. For even if the weak component were as brilliant as the intense component, the light is only used in the author's form of interferometer to produce interference bands of a very low order of interference, formed by the interfering reflections from two glass surfaces only about one or at most two millimetres apart, and not for high orders of interference such as are afforded and employed in the long étalons of MICHELSON and FABRY and PEROT.

If we use this value 6562·869, along with the standard Cd_r wave-length, to calculate

* 'Proc. Roy. Soc.,' A, vol. 90, p. 605 (1914).

† 'Phys. Rev.,' vol. 30, p. 608 (1927).

‡ 'Proc. Roy. Soc.,' A, vol. 97, p. 307 (1920).

§ 'Phil. Trans.,' A, vol. 217, p. 237 (1917).

the number of Cd_r wave-lengths in the yard, we obtain the number 1420210·6, in remarkable agreement with the neon result.

But it may be objected with some ground, however, that to accept for the value of the wave-length of H_a, that derived from the determination of the number of interference bands (for H_a) in the yard, and necessarily converted (even although it be by use of the best available yard : metre ratio, confirmed by the neon result) to metric measure (this measure being used for light wave-lengths), savours of working in a circle. It should be remembered, however, that the International Committee have not only accepted BENOIT, FABRY and PEROT's determination of the number of interference fringes of Cd_r (and therefore of the half, the number of wave-lengths) in the metre as affording the standard length of the international metre, but as affording also the primary standard wave-length of light, that of Cd_r. Supposing, however, we entertain the objection, the next best course is obviously to use the most trustworthy value derived from other recent work, namely, that of HOUSTON for the brighter component (predominatingly acting as shown below) of the doublet, 6562·8473. As he himself states that this value is too low when referred to the cadmium standard, the true value must be somewhat higher. When it is thus used (p. 312) to calculate the number of wave-lengths of red cadmium light in the yard, the result is 1420205·9, as shown on p. 312, $4\frac{1}{2}$ wave-lengths less than when the author's value is used directly. Or we can employ the author's neon result to calculate the wave-length of H_a, from the facts that the yard contains 1393290·5 H_a wave-lengths and 1562408·6 Ne_c wave-lengths, and using the standard Ne_c wave-length 5852·488 Å.U. This affords the value for H_a 6562·865 Å.U These are the most probable alternatives when the author's result is excluded. For it is clear that the brighter component of the H_a doublet is responsible for the bands at the conclusion of the base-line count, the intensity having diminished to eventual feebleness with increasing separation of the two reflecting surfaces up to $2\frac{1}{2}$ mm. If, however, we use the value for the "optical centre of gravity" of the doublet, that of CURTIS 6562·793, we obtain the number 1420194·1 for the Cd_r wave-lengths in the yard, 15·7 wave-lengths lower than that derived from the neon determination. The mean result of the neon and hydrogen determinations would then be 1420202·0 wave-lengths of Cd_r in the yard, supposing we assign an equal value to the Ne_c and H_a determinations. But the author emphatically attributes a higher value to the neon result, and believes that the value 1420209·8 derived from it is very near the truth.

Concluding Summary.

The British Imperial Standard Yard has been evaluated in wave-lengths of the red hydrogen line H_a, the yellow line of neon Ne_c, and the red line of cadmium Cd_r, the latter by calculation from the experimental results with hydrogen and neon. The interferential method of the author has been employed, involving the combined use of GRAYSON-ruling fiducial marks of only half-wave-length thickness, the author's interferometer, and the TUTTON comparator at the Standards Department, a method totally different from that of MICHELSON and that of BENOIT, FABRY and PEROT, by which they evaluated the International metre in wave-lengths of cadmium red light. The author has discarded direct determinations in cadmium light, as the necessity of heating the cadmium vacuum tube to 330° C. seriously interfered with an accurate result. The results calculated from the hydrogen and neon determinations agreed within fairly close limits, which are especially satisfactory when the relative coarseness of the defining lines on the Imperial yard bar (22 wave-lengths in thickness) is considered.

The results are, that the Imperial Standard Yard contains, at 62° F. (the official standard temperature) and 760 mm. pressure :—

(*a*) 1393290·5 wave-lengths of red hydrogen light (H_α).

(*b*) 1562408·6 wave-lengths of yellow neon light (Ne_c).

(*c*) 1420209·8 [calculated from (*b*)] wave-lengths of red cadmium light (Cd_r)
1420210·6 [calculated from (*a*)] wave-lengths of red cadmium light (Cd_r)
or 1420205·9 if HOUSTON's value for the wave-length of H_α be used instead of the author's.

The neon result is quite free from any ambiguity whatever, the wave-length of the unresolvable line Ne_c compared with the cadmium standard being very accurately known, as indeed a secondary standard. The author therefore believes the neon result to be the nearest to the truth, in indicating 1420209·8 wave-lengths of red cadmium light in the Imperial Yard, and suggests that this value, or the round number 1420210, be taken as the final result of this investigation, the H_α results being only regarded as affording general confirmation, the ambiguity as to its wave-length (on account of H_α not being a single line but a close doublet) rendering it not so absolutely trustworthy.

It is suggested, therefore, that the round number 1420210 of wave-lengths of red cadmium light, of wave-length 6438·4698 Å.U., the standard wave-length now generally recognised (the mean of the values of MICHELSON and of BENOIT, FABRY and PEROT), might be taken as the recognised length of the British Imperial Standard Yard at 62° F. and 760 mm. pressure.

The relation between the yard and the metre, as determined non-interferometrically, by SEARS, JOHNSON and JOLLY, namely 1 yard = 0·91439841 metre, and the mean of MICHELSON's and BENOIT, FABRY and PEROT's values of the metre, 1553163·82 wave-lengths of Cd_r, can be used to calculate the number of wave-lengths of Cd_r in the yard. The result afforded by doing so is 1420210·5, a most satisfactory concordance with the number now communicated.

Perhaps the most interesting fact about this investigation is that the author has been able to measure a base-line of $\frac{1}{16}$-inch, between GRAYSON-ruling fiducial limit-marks of less than a half wave-length thickness, by direct counting of the interference bands, 5423 in neon light, which are comprised in this interval. Indeed, as this has been done in duplicate, on two different sets of rulings, and the second has been transferred by the comparator alongside the first, on to the adjacent (second) $\frac{1}{16}$-inch, the base-line has in reality been the 10,846 bands in neon light (and corresponding numbers for hydrogen light) contained in the $\frac{1}{8}$-inch, all directly counted, without any adventitious aids whatever. The maximum counted by MICHELSON was 1626, as base-line for the metre.

The author desires to offer his sincere thanks to the Deputy Warden of the Standards, J. E. SEARS, Esq., C.B.E., for the facilities so generously afforded for employing the TUTTON comparator at the Standards Department of the Board of Trade, and for the use of the Standard Yard of 1926, and the invar bar No. 27 of the National Physical Laboratory. Also to R. J. TRUMP, Esq., Chief Assistant at the Standards Department, for very ably supervising the working of the electric thermostat which maintains the comparator room at the official standard temperature of 62° F. Further, to CONRAD BECK, Esq., for the loan of some especially excellent high-power objectives and eye-pieces for the comparator microscopes, to render them as suitable for this work as those provided with the author's interferometer.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES